

## Problem of the Week

### Problem C and Solution

#### Stack 'Em Up

#### Problem

Three cubes with side lengths 1 m, 2 m and 3 m are stacked on top of each other as shown.

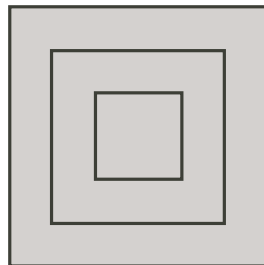
Determine the total surface area of the stack, including the bottom.

#### Solution

To determine the areas we will primarily use  $Area = length \times width$ .

Each cube has four exposed square sides so the total area of all the sides is  $4 \times (1 \times 1) + 4 \times (2 \times 2) + 4 \times (3 \times 3) = 4 \times (1) + 4 \times (4) + 4 \times (9) = 4 + 16 + 36 = 56 \text{ m}^2$ .

To determine the exposed top area of each of the cubes look down on the tower and see a cross-section like the one below.



This exposed area is exactly the same as the side area of one face of the largest cube. Therefore, the top exposed area is  $3 \times 3 = 9 \text{ m}^2$ . The top area and the bottom area are the same. Therefore, the bottom area is  $9 \text{ m}^2$ .

**The total surface area is  $56 + 9 + 9 = 74 \text{ m}^2$ .**

**Extension:** Three cubes with side lengths  $x$ ,  $y$  and  $z$  are stacked on top of each other in a similar manner to the original problem such that  $0 < x < y < z$ . Show that the total surface area of the stack, including the bottom, is  $6z^2 + 4y^2 + 4x^2$ .

